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Turbulence model for flow through porous media

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Abstract—The macroscopic governing equations for the turbulent flow through porous media consisting of packed spheres are examined and the 0-equation model is proposed. In the process of deriving the macroscopic governing equations by using the local volume averaging technique, we consider the effective eddy diffusivity as the algebraic sum of the eddy diffusivities estimated from two types of vortices: (i) the pseudo vortex of the order of the particle diameter, and (ii) the interstitial vortex between the solid particles. Furthermore, it is shown that the 0-equation model proposed in this study can predict the flow and heat transfer characteristics at high Reynolds number. Copyright © 1996 Elsevier Science Ltd.

INTRODUCTION

Examining a turbulence model for the flow and heat transfer characteristics in a turbulent field through porous media is of importance in such a prediction of the thermal dispersion in packed beds. The macroscopic conservation equations in porous media can be obtained by locally averaging microscopic conservation equations over a representative volume [1], and a physical quantity is related with the local volume average of fluid phase or solid phase. Likewise, we have to construct the turbulence model which reflects the microscopic vortex behaviors intrinsic to porous media. Thus we should notice that the eddy viscosity for porous media has a close relevance to the microscopic turbulence mixing and the geometric structure in porous media.

The Forchheimer flow resistance and dispersion, which are the phenomena observed in the flow through porous media at high Reynolds number, have been interpreted by using the concept of laminar flow theory [e.g. 2–9]. Vafai and Tien [2] considered the Forchheimer flow resistance as an inertial effect of flow offered through the solid matrix (form drag). Many models of the dispersion have been derived from the correlation term between the spatial fluctuations of the velocity and temperature (or concentration) quantities. On the other hand, there is an interesting report [10] in which the turbulence vortices begin to appear at $Re_d \approx 10$ and gradually cover the flow domain (pore space) as Reynolds number increases, and the velocity measurements with a hot-wire anemometer confirm the existence of turbulence in packed beds [11, 12]. In addition to these reports, there exist many experiments [13–20] in which the deviation from Darcy's law is observed at $Re_d \approx 10$ and not only the effect of the Forchheimer flow resistance, but also the effect of the dispersion, gradually become predominant as the Reynolds number increases. Judging from the above, it seems reasonable to suppose that the Forchheimer flow resistance and the dispersion are caused mainly by turbulent mixing (diffusion) in porous media. Lee and Howell [21] proposed the k-ε model for flow through porous media with high porosity and considered the same eddy viscosity for porous media as one which is commonly used for the pure fluid. Travkin *et al.* [22] developed the turbulence model in highly porous media along with a statistical and numerical methodology.

In this study, examining the behavior of the microscopic turbulent field intrinsic to porous media, we construct the macroscopic governing equations for turbulent flow through porous media consisting of packed spheres and clarify the relation between the momentum and energy transports due to the turbulent vortex mixing at high Reynolds number. The closure schemes for the Reynolds stress tensor and the turbulent heat flux vector are obtained based on the eddy viscosities and the eddy thermal conductivities estimated from two types of vortices. Furthermore, a 0-equation model for the eddy diffusivity is proposed and it is shown that the Forchheimer flow resistance and the thermal dispersion can be explained from the present 0-equation model.

MOMENTUM EQUATION

We present a brief discussion on local volume averaging by Slattery [1] (see Fig. 1). If B is any scalar, spatial vector or second-order tensor associated with the fluid-phase, the local volume average over V_f of a quantity B associated with the fluid-phase is defined as

$$\langle B \rangle^{(f)} \equiv \frac{1}{V_f}\int_{V_f} B \, \mathrm{d}V \tag{1}$$

NOMENCLATURE

A_w	interfacial area between fluid and solid phases in Fig. 1
c	specific heat
d_p	particle diameter
$\mathbf{I}$	identity tensor
k	turbulent kinetic energy
K	permeability
P	pressure
Pe	Peclet number defined in equation (47)
Pe_d	particle Peclet number defined in equation (52)
Pr_t	turbulent Prandtl number
$\mathbf{q}$	heat flux vector
Re_d	particle Reynolds number
$\mathbf{S}$	molecular stress tensor defined in equation (6)
$\mathbf{S}_t$	turbulent stress tensor defined in equation (7)
t	time
$\mathbf{U}$	time-averaged microscopic velocity vector
V	elementary volume.

Greek symbols

α	thermal diffusivity $\equiv \lambda_f/(\rho_f c_f)$
ϕ	porosity
γ	eddy viscosity ratio defined in equation (43)
λ_f	thermal conductivity of fluid phase
λ_e	effective thermal conductivity of porous medium saturated with stagnant fluid [23]
λ_p	thermal dispersion defined in equation (35)
λ_s	thermal conductivity of solid phase
λ_t	eddy (turbulent) thermal conductivity defined in equation (27)
$\lambda_{t,P}$	pseudo eddy thermal conductivity
$\lambda_{t,V}$	void eddy thermal conductivity
μ	viscosity
μ_t	eddy (turbulent) viscosity in equation (13)
$\mu_{t,P}$	pseudo eddy viscosity
$\mu_{t,V}$	void eddy viscosity
ρ	density
σ	correction factor defined in equation (14)
ξ	thermal conductivity ratio $\equiv \lambda_s/\lambda_f$.

Subscripts

f	associated with fluid phase
s	associated with solid phase.

and the local volume average over V_s of a quantity B associated with the solid-phase is defined as

$$\langle B\rangle^{(s)} \equiv \frac{1}{V_s}\int_{V_s} B\,\mathrm{d}V. \tag{2}$$

Furthermore, the theorems for the volume average of a gradient and a divergence is expressed by equations (3) and (4), respectively

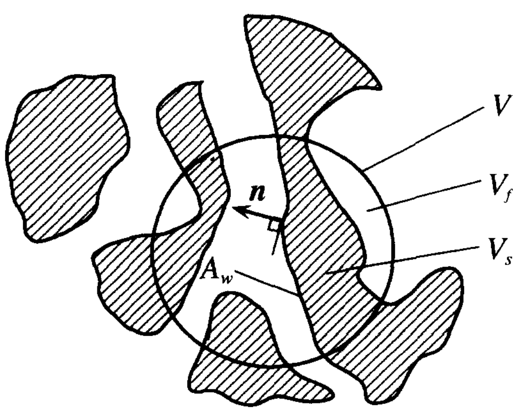

Fig. 1. Control volume of local volume averaging for porous structure.

$$\langle \nabla B\rangle^{(f)} = \nabla\langle B\rangle^{(f)} + \frac{1}{V_f}\int_{A_w} B\mathbf{n}\,\mathrm{d}A \tag{3}$$

$$\langle \operatorname{div}\mathbf{B}\rangle^{(f)} = \operatorname{div}\langle \mathbf{B}\rangle^{(f)} + \frac{1}{V_f}\int_{A_w} \mathbf{B}\cdot\mathbf{n}\,\mathrm{d}A \tag{4}$$

where A_w is the area of the interface between the fluid and solid phase in $V(=V_f+V_s)$. We will derive the macroscopic governing equations for turbulent flow through porous media by the above local volume averaging technique.

In the turbulent flow through porous media, the microscopic momentum equation can be given by the Reynolds equation coupled with Boussinesq's eddy viscosity formulation

$$\rho_f\left[\frac{\partial \mathbf{U}}{\partial t} + \operatorname{div}(\mathbf{UU})\right] = \operatorname{div}(\mathbf{S}+\mathbf{S}_t) \tag{5}$$

where

$$\mathbf{S} = 2\mu\mathbf{D} - P\mathbf{I} \tag{6}$$

$$\mathbf{S}_t = 2\mu_t\mathbf{D} - \tfrac{2}{3}k\mathbf{I} \tag{7}$$

$$\mathbf{D} = \tfrac{1}{2}[\nabla\mathbf{U} + (\nabla\mathbf{U})^{\mathrm{T}}]. \tag{8}$$

If the eddy viscosity μ_t is assumed to be constant

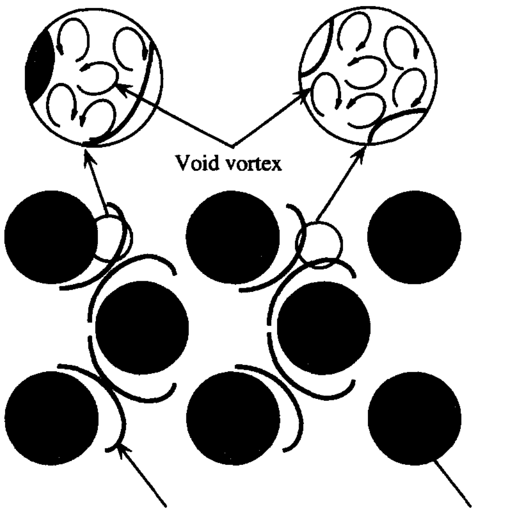

Fig. 2. Schematic model of vortices in packed beds.

within V_f, the local volume average of equation (5) leads to the macroscopic Reynolds equation for the turbulent flow through porous media.

$$\rho_f\left[\frac{\partial\langle \mathbf{U}\rangle^{(f)}}{\partial t}+\langle \mathrm{div}\,(\mathbf{UU})\rangle^{(f)}\right]=\langle \mathrm{div}\,(\mathbf{S}+\mathbf{S}_t)\rangle^{(f)}. \tag{9}$$

The drag forces around the solid particles can be derived from the right hand side of the above equation with the aid of equation (4).

$$\langle \mathrm{div}\,\mathbf{S}\rangle^{(f)}=\mathrm{div}\,\langle \mathbf{S}\rangle^{(f)}+\frac{1}{V_f}\int_{A_w}\mathbf{S}\cdot\mathbf{n}\,\mathrm{d}A \tag{10}$$

$$\langle \mathrm{div}\,\mathbf{S}_t\rangle^{(f)}=\mathrm{div}\,\langle \mathbf{S}_t\rangle^{(f)}+\frac{1}{V_f}\int_{A_w}\mathbf{S}_t\cdot\mathbf{n}\,\mathrm{d}A. \tag{11}$$

The second term on the right hand side of equation (10) is the drag force caused by the molecular stress tensor $\mathbf{S}$ and that of equation (11) is the drag force caused by the Reynolds (turbulent) stress tensor $\mathbf{S}_t$. We recognize that the second term on the right hand side of equation (10) is the original Darcy flow resistance.

$$\frac{1}{V_f}\int_{A_w}\mathbf{S}\cdot\mathbf{n}\,\mathrm{d}A=-\phi\frac{\mu}{K}\langle \mathbf{U}\rangle^{(f)}. \tag{12}$$

Whereas, Vafai and Tien [2] formulate the above equation by a linear combination of Darcy's flow resistance and Forchheimer's flow resistance, we consider Forchheimer's flow resistance to be relevant to the drag force due to the turbulent mixing.

We now discuss the Reynolds stress tensor $\mathbf{S}_t$. As shown in Fig. 2, it is expected that two types of vortices, namely the void and pseudo vortices, play an important role in the transport mechanism of the turbulent flow through porous media. Taking notice of the flow along the solid particles, we can suppose that there arises the forced flow distortion due to the interruption of the solid particles. The flow distortion will transport the fluid lump far away and cause the associated exchange of momenta. So we refer to this momentum diffusion as the mixing of the pseudo vortex. The void vortex is the interstitial vortex which is formed in the pore between the solid particles. It can be estimated that the characteristic length scale of the pseudo vortex is the order of magnitude of the particle diameter d_p and that of the interstitial vortex is of the gap width $\sqrt{K}$. Thus we consider the eddy viscosity μ_t in equation (7) as the algebraic sum of the eddy viscosities defined by the characteristic length scales of the pseudo and void vortices:

$$\mu_t=\mu_{t,P}+\mu_{t,V} \tag{13}$$

where the first term on the right hand side of equation (13) is the pseudo eddy viscosity $\mu_{t,P}$ characterized by the pseudo vortex and the second term is the void eddy viscosity $\mu_{t,V}$ by the void vortex. It is fair to say that the Reynolds stress tensor related to the void eddy viscosity, which is characterized by the interstitial vortices, contributes toward the drag force, because the pseudo vortex takes a role of the long-distance momentum transport owing to the forced flow distortion, while the void vortex directly determines the velocity profile of the turbulent shear flow along the solid particle due to the effect of its short-distance momentum exchange. Furthermore, equation (11) reduces to equation (14) on the grounds that the drag force caused by the molecular stress $\mathbf{S}$ tensor is expressed by equation (12)

$$\langle \mathrm{div}\,\mathbf{S}_t\rangle^{(f)}=\mathrm{div}\,\langle \mathbf{S}_t\rangle^{(f)}-\sigma\phi\frac{\mu_{t,V}}{K}\langle \mathbf{U}\rangle^{(f)} \tag{14}$$

where σ is the correction factor which is introduced to extend the concept of the hydrodynamic conductance defined by Darcy's law to the turbulent flow. We estimate the correction factor as $\sigma\sim 1$ by considering the similar contributions of the turbulent kinetic energy k and the pressure P to the stress tensors defined by equations (6) and (7). Here we may note that the second term on the right hand of equation (14) represents the damping effect due to the void vortex associated with the local homogeneous and isotropic effects of turbulence.

Next we shall concentrate on the inertial term of equation (9). The microscopic velocity vector $\mathbf{U}$ can decompose into the sum of the mean velocity vector $\langle \mathbf{U}\rangle^{(f)}$ (spatial average) and the deviation velocity vector $\mathbf{u}^V$ (spatial fluctuation), i.e.

$$\langle \mathbf{U}\rangle\equiv\frac{1}{V_f}\int_{V_f}\mathbf{U}\,\mathrm{d}V+\mathbf{u}^V=\langle \mathbf{U}\rangle^{(f)}+\mathbf{u}^V. \tag{15}$$

Substituting equation (15) into the inertial term of equation (9) gives

$$\mathrm{div}\,\langle \mathbf{UU}\rangle^{(f)} = \mathrm{div}\,[\langle \mathbf{U}\rangle^{(f)}\langle \mathbf{U}\rangle^{(f)}] + \mathrm{div}\,\langle \mathbf{u}^{\mathrm{V}}\mathbf{u}^{\mathrm{V}}\rangle^{(f)}. \quad (16)$$

The porous structures are commonly held periodic in the representative length scale related to the representative volume V, so that the spatial fluctuation may be considered as the almost periodic function of the representative length scale. As the divergence operator is valid for the representative length scale in principle, it is supposed that the second term on the right hand of equation (16) is negligible due to its periodic nature as compared to the first term. Therefore, the inertial term can be approximated by

$$\mathrm{div}\,\langle \mathbf{UU}\rangle^{(f)} = \mathrm{div}\,[\langle \mathbf{U}\rangle^{(f)}\langle \mathbf{U}\rangle^{(f)}]. \quad (17)$$

By the above closure modeling for the drag force and the inertia, the macroscopic momentum equation for the turbulent flow through porous media becomes

$$\rho_f\left[\frac{\partial\langle \mathbf{U}\rangle^{(f)}}{\partial t} + \mathrm{div}\,\{\langle \mathbf{U}\rangle^{(f)}\langle \mathbf{U}\rangle^{(f)}\}\right] = \mathrm{div}\,\langle \mathbf{S}+\mathbf{S}_t\rangle^{(f)} - \phi\frac{\mu+\sigma\mu_{t,\mathrm{V}}}{K}\langle \mathbf{U}\rangle^{(f)}. \quad (18)$$

ENERGY EQUATION

The microscopic energy equations for the fluid and solid phases are given by

$$\rho_f c_f\frac{\partial T}{\partial t} + \rho_f c_f\,\mathrm{div}\,(\mathbf{U}T) + \mathrm{div}\,\mathbf{q}_f = 0 \quad (19)$$

$$\rho_s c_s\frac{\partial T}{\partial t} + \mathrm{div}\,\mathbf{q}_s = 0 \quad (20)$$

where the heat flux vectors for the fluid and solid phases are

$$\mathbf{q}_f = -(\lambda_f+\lambda_t)\nabla T \quad (21)$$

$$\mathbf{q}_s = -\lambda_s\nabla T. \quad (22)$$

With the aid of equations (1) and (2), the local volume averages of equations (19) and (20) yield the macroscopic energy equations for the fluid and solid phases.

$$\rho_f c_f\frac{\partial\langle T\rangle^{(f)}}{\partial t} + \rho_f c_f\,\mathrm{div}\,\langle \mathbf{U}T\rangle^{(f)} + \mathrm{div}\,\langle \mathbf{q}_f\rangle^{(f)} + \frac{1}{V_f}\int_{A_w}\mathbf{q}_f\cdot\mathbf{n}\,\mathrm{d}A = 0 \quad (23)$$

$$\rho_s c_s\frac{\partial\langle T\rangle^{(s)}}{\partial t} + \mathrm{div}\,\langle \mathbf{q}_s\rangle^{(s)} - \frac{1}{V_s}\int_{A_w}\mathbf{q}_s\cdot\mathbf{n}\,\mathrm{d}A = 0. \quad (24)$$

As the heat flux vector is continuous at the interface between the fluid and solid phases, the interface condition is given by

$$\int_{A_w}\mathbf{q}_f\cdot\mathbf{n}\,\mathrm{d}A = \int_{A_w}\mathbf{q}_s\cdot\mathbf{n}\,\mathrm{d}A. \quad (25)$$

Substituting equations (23) and (24) into the above interface condition leads to

$$\phi\left[\rho_f c_f\left\{\frac{\partial\langle T\rangle^{(f)}}{\partial t} + \mathrm{div}\,\langle \mathbf{U}T\rangle^{(f)}\right\} + \mathrm{div}\,\langle \mathbf{q}_f\rangle^{(f)}\right] + (1-\phi)\left[\rho_s c_s\frac{\partial\langle T\rangle^{(s)}}{\partial t} + \mathrm{div}\,\langle \mathbf{q}_s\rangle^{(s)}\right] = 0. \quad (26)$$

We shall now focus on the eddy thermal conductivity. Suppose that an analogy exists between the eddy viscosity and the eddy thermal conductivity, the eddy thermal conductivity λ_t can be represented as the algebraic sum of the eddy thermal conductivities defined by the characteristic length scales of the pseudo and void vortices:

$$\lambda_t = \lambda_{t,\mathrm{P}} + \lambda_{t,\mathrm{V}} \quad (27)$$

where the first term on the right hand side of the above equation is the pseudo eddy thermal conductivity $\lambda_{t,\mathrm{P}}$ characterized by the pseudo vortex and the second term is the void eddy thermal conductivity $\lambda_{t,\mathrm{V}}$ by the void vortex. The pseudo vortex contributes to the long-distance heat transport due to the forced flow distortion, so that the thermal tortuosity cannot be produced from the heat flux vector related to the pseudo eddy thermal conductivity. Accordingly, the heat flux vectors can be reduced to equations (28) and (29) with the aid of equations (1) and (2):

$$\langle \mathbf{q}_f\rangle^{(f)} = -(\lambda_f+\lambda_t)\nabla\langle T\rangle^{(f)} - \frac{\lambda_f+\lambda_{t,\mathrm{V}}}{V_f}\int_{A_w}T\mathbf{n}\,\mathrm{d}A \quad (28)$$

$$\langle \mathbf{q}_s\rangle^{(s)} = -\lambda_s\nabla\langle T\rangle^{(s)} + \frac{\lambda_s}{V_s}\int_{A_w}T\mathbf{n}\,\mathrm{d}A. \quad (29)$$

We define the local volume average of temperature over the fluid and solid phases

$$\langle T\rangle^{(m)} \equiv \frac{1}{V}\int_V T\,\mathrm{d}V = \phi\langle T\rangle^{(f)} + (1-\phi)\langle T\rangle^{(s)} \quad (30)$$

and we make use of the local thermal equilibrium assumption [1]

$$\langle T\rangle^{(m)} = \langle T\rangle^{(f)} = \langle T\rangle^{(s)}. \quad (31)$$

As we have mentioned in the preceding section, the spatial fluctuation may be considered as the almost periodic function of the representative length scale since the porous structures are commonly held periodic. Therefore the correlation term between the spatial fluctuations of the velocity and temperature quantities, which has been related to the thermal dispersion effect in the previous studies [5–9], can be neglected and the enthalpy transport term in equation (26) can be approximated by

$$\operatorname{div}\langle \mathbf{U}T\rangle^{(f)} = \operatorname{div}[\langle \mathbf{U}\rangle^{(f)}\langle T\rangle^{(m)}]. \quad (32)$$

Furthermore, introducing the concept of the effective thermal conductivity proposed by Kunii and Smith [23] leads to

$$\lambda_e \nabla\langle T\rangle^{(m)} = \{\phi\lambda_f + (1-\phi)\lambda_s\}\nabla\langle T\rangle^{(m)} + \frac{\lambda_f - \lambda_s}{V}\int_{A_w} T\mathbf{n}\,\mathrm{d}A. \quad (33)$$

By the above closure modeling for the turbulent heat flux and enthalpy transport terms, the macroscopic energy equation for the turbulent flow through porous media becomes

$$[\phi\rho_f c_f + (1-\phi)\rho_s c_s]\frac{\partial\langle T\rangle^{(m)}}{\partial t} + \phi\rho_f c_f \operatorname{div}[\langle \mathbf{U}\rangle^{(f)}\langle T\rangle^{(m)}] = \operatorname{div}[\lambda_p \nabla\langle T\rangle^{(m)}] \quad (34)$$

where

$$\lambda_p = \lambda_e + \phi\lambda_t + f_t\lambda_{t,V} \quad (35)$$

$$f_t = \frac{\lambda_e - \{\phi\lambda_f + (1-\phi)\lambda_s\}}{\lambda_f - \lambda_s}. \quad (36)$$

0-EQUATION MODEL

In this section, we propose the 0-equation model for the eddy viscosity and the eddy thermal conductivity in the macroscopic momentum and energy equations which have been constructed in the foregoing sections. From equation (18), the momentum equation in the fully developed one-dimensional turbulent flow is represented by

$$-\frac{\partial P}{\partial x} = \sigma\frac{\mu_{t,V}}{K}U. \quad (37)$$

With the aid of the Kolmogorov–Prandtl expression [24], the void eddy viscosity $\mu_{t,V}$ becomes

$$\mu_{t,V} = C_\mu \rho_f \sqrt{k}L. \quad (38)$$

If the velocity scale $\sqrt{k}$ is of order $\sqrt{\overline{U^2}}$ and the length scale L is of order $\sqrt{K}$, the void eddy viscosity $\mu_{t,V}$ is estimated as

$$\mu_{t,V} \sim \rho_f\sqrt{\overline{U^2}}\sqrt{K}. \quad (39)$$

On the other hand, the empirical correlation for the flow resistance of packed beds at high Reynolds number [13] is given by

$$-\frac{\partial P}{\partial x} = F\frac{\rho_f U^2}{\sqrt{K}} \quad (40)$$

$$F = \frac{1.75}{\sqrt{150\phi^3}} \quad \text{(packed bed)}. \quad (41)$$

Equation (40) is the so-called Forchheimer flow resistance. Comparing equations (37) and (40), we write the void eddy viscosity $\mu_{t,V}$ as

$$\mu_{t,V} = \frac{F}{\sigma}\rho_f\sqrt{\overline{U^2}}\sqrt{K}. \quad (42)$$

It is noteworthy that the void eddy viscosity obtained from the Kolmogorov–Prandtl expression [equation (39)] is the same as from the empirical correlation. From this fact, we may deduce the Forchheimer flow resistance from the mixing (diffusion) process to which the void vortex contributes.

We focus on the mutual relation between the void and pseudo eddy viscosities (thermal conductivities). The eddy viscosity ratio γ is defined as

$$\gamma = \frac{\mu_{t,P}}{\mu_{t,V}}. \quad (43)$$

By introducing the turbulent Prandtl number Pr_t which is assumed to be independent of the vortex length scale, the void and pseudo eddy thermal conductivities are

$$\lambda_{t,P} = \frac{c_f}{Pr_t}\mu_{t,P} \quad (44)$$

$$\lambda_{t,V} = \frac{c_f}{Pr_t}\mu_{t,V}. \quad (45)$$

With the aid of equations (43)–(45), equation (35) can be rewritten as

$$\frac{\lambda_p}{\lambda_f} = \frac{\lambda_e}{\lambda_f} + \phi\frac{\lambda_t}{\lambda_f} + f_t\frac{\lambda_{t,V}}{\lambda_f} = \frac{\lambda_e}{\lambda_f} + \frac{F\{\phi(\gamma+1)+f_t\}}{\sigma Pr_t}Pe \quad (46)$$

where

$$Pe = \frac{\sqrt{\overline{U^2}}\sqrt{K}}{\alpha}. \quad (47)$$

If the turbulent Prandtl number Pr_t is independent of the Peclet number Pe, the relationship [7, 9, 14] where the thermal dispersion λ_p is proportional to the Peclet number Pe is derived from equation (46). Furthermore, with the aid of the Blake and Kozeny expression [13] for the permeability of packed beds

$$K = \frac{\phi^3 d_p^2}{150(1-\phi)^2} \quad (48)$$

the eddy viscosity ratio γ is estimated as

$$\gamma \sim \frac{d_p}{\sqrt{K}} \approx 30 \quad (49)$$

where the porosity is treated as 0.4.

The contribution of the eddy viscosity ratio γ to the

Table 1. Relationship between the thermal conductivity ratio ξ and the coefficient f_t

ξ	0.01	0.1	1	10	100
f_t	1.02×10^{-3}	9.37×10^{-3}	2.00×10^{-1}	2.93×10^{-1}	5.15×10^{-1}

thermal dispersion is shown in Fig. 3 with the empirical correlation obtained by Yagi *et al.* [14]

$$\frac{\lambda_p}{\lambda_f} = 7.5+0.8Pe_d \quad \text{(glass spheres)} \tag{50}$$

$$\frac{\lambda_p}{\lambda_f} = 13+0.7Pe_d \quad \text{(steel spheres)} \tag{51}$$

where the particle Peclet number Pe_d is

$$Pe_d = Pe\frac{d_p}{\sqrt{K}} = \frac{\sqrt{U^2}d_p}{\alpha}. \tag{52}$$

It can be seen from Fig. 3 that the increase of the eddy viscosity ratio enhances the thermal dispersion and that such a tendency is remarkable in the region of high particle Peclet number. The present result of $\gamma = 100$ is in good agreement with the empirical data of Yagi *et al.* Though this value of the eddy viscosity ratio $\gamma = 100$ is somewhat greater than the value estimated from equation (49), both can be considered to be of the same order of magnitude.

Table 1 indicates the relationship between the thermal conductivity ratio ξ and the coefficient f_t defined in equation (36) at the porosity $\phi = 0.4$. The coefficient f_t is very small compared with the eddy viscosity ratio $\gamma = 100$. Judging from this fact, the contribution of the void eddy thermal conductivity to the thermal dispersion is negligible. In other words, the mixing of the pseudo vortex mainly contributes to the thermal dispersion and equation (46) reduces to

$$\frac{\lambda_p}{\lambda_f} = \frac{\lambda_e}{\lambda_f} + \frac{F\phi\gamma}{\sigma Pr_t}Pe. \tag{53}$$

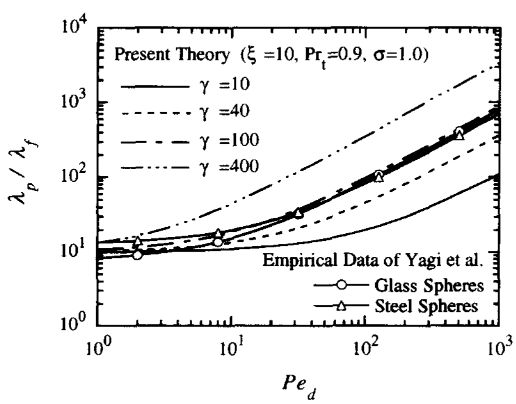

Fig. 3. Comparison of the present model with the empirical data of Yagi *et al.* [14].

CONCLUDING REMARKS

This study proposes a model of vortex transports of turbulent flow through porous media, comparing the theoretical results with the previous experimental results. The following conclusions are derived:

(1) The momentum and energy transports in the turbulent flow through porous media can be explained rationally by introducing the concept of two types of vortices. One is the interstitial (void) vortex of the order of the thickness of the gap width $\sqrt{K}$ which is formed in the pore between particles and the other is the pseudo vortex of the order of the particle diameter d_p which reflects the forced flow distortion due to the interruption of the solid particles.

(2) Considering the eddy diffusivity as the algebraic sum of the eddy diffusivities defined by the characteristic length scales of the pseudo vortex and the void vortex, we have constructed the macroscopic momentum and energy equations for turbulent flow through porous media.

(3) The Forchheimer flow resistance and the thermal dispersion which are reported in the previous experiments at high Reynolds number are well described by the present 0-equation model, and it is clarified that the mixing of the void vortex mainly contributes to the Forchheimer flow resistance and the mixing of the pseudo vortex to the thermal dispersion.

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